

$$\begin{array}{r}
 A B C D \\
 9 \\
 \hline
 D C B A \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 C \\
 C A C \\
 + A C B C \\
 \hline
 2 A A C
 \end{array}$$

Cryptarithm

Math Olympiad Training



Introduction

- Cryptarithm is also known as Alphametrics
- Each digit is represented an alphabet.

Examples

- 5 can be represented by A
- 423 can be represented by ABC where $A = 4$, $B = 2$ and $C = 3$
- AAB if A is 8 then the 3-digit number is 88B where B can be 0 to 9 but not 8.



IMPORTANT PRINCIPLES

Sum of two digits must be from **0 to 18**.

- Largest digit is 9 => $9 + 9 = 18$
- Smallest digit is 0 => $0 + 0 = 0$

Hence

To have a carry over $A + B \geq 10$

E.g. $4 + 6 = 10$ or $5 + 5 = 10$

If $A + A$ is 2-digit and the last digit is 0 then A has to be 5.

If $A + A$ is 2-digit and last digit is 8 then A has to be 9.

Tips:

$$5 + 5 = 10$$

$$6 + 6 = 12$$

$$7 + 7 = 14$$

$$8 + 8 = 16$$

$$9 + 9 = 18$$



IMPORTANT PRINCIPLES

The product of two digits must range from 0 to 81.

- Largest digit is 9 => $9 \times 9 = 81$
- Smallest digit is 0 => $0 \times 0 = 0$

Carry over of multiplication can be > 1 unlike addition which is always 1.

E.g. $8 \times 8 = 64$ so the carry is 6.

$A \times B = 2A$ which means the second digit is A.

Final answer needs to be between 20 to 29.

You can guess A is 4 and B is 6 since $4 \times 6 = 24$.

There is no other answer other possible pairs that give 20 to 29 can't satisfy.

Other possibilities:

$3 \times 7 = 21$, $3 \times 8 = 24$, $3 \times 9 = 27$, $4 \times 5 = 20$, $4 \times 7 = 28$, $5 \times 4 = 20$, $5 \times 5 = 25$



1. What digit does each letter (A, B, C) represent?

$$\begin{array}{r}
 A\ B \\
 A\ B \\
 + \underline{A\ B} \\
 C\ C\ C
 \end{array}$$

Solution

AB are similar two-digit numbers.

C cannot be 3 or greater because 333 divided by 3 will result in 3-digit number and not 2-digit.

If C = 1,

$$\begin{array}{l}
 111 / 3 = 37 \\
 A = 3 \text{ and } B = 7
 \end{array}$$

If C = 2,

$$\begin{array}{l}
 222 / 3 = 74 \\
 A = 7 \text{ and } B = 4
 \end{array}$$



2. What does each digit represent?

a)

$$\begin{array}{r} \\ \\ \hline x \\ \hline \end{array}$$

Solution

Since $A \times A$ gives 10, then guess A should be 3.
And there is a carry of 1 to give 10.

$$3 \times 3 = 9$$

If A is 3 then what number $\times 3$ will give the same digit?

C should be 5 since $5 \times 3 = 15$ and gives a carry of 1.

$B \times A + 1$ will give 3 and a 2-digit number.

B should be 4 since $4 \times 3 = 12$ and $12 + 1 = 13$

$A = 3, B = 4$ and $C = 5$

b)

$$\begin{array}{r} \\ \\ \hline x \\ \hline \end{array}$$

Solution

Since $A \times 9$ is a single digit then A must be 1 since if $A > 1$ will give 2- digit result

$$\text{E.g. } 2 \times 9 = 18$$

D must be 9 since $9 \times 9 = 81$ and no other digit will give "1" for last digit.

$$1 $$

$$x $$

$$9 $$

$B \times 9$ has no carry, so $B = 0$ as it can't be 1.

$C \times 9 + 8$ gives 0 as second digit and carry is also C.

$C = 8$ since $8 \times 9 + 8 = 80$ and $0 \times 9 + 8 = 8$

$A = 1, B = 0, C = 8, D = 9$



3. What does each digit represent?

a)

$$\begin{array}{r} \\ \\ + \\ \hline 1 9 8 9 \end{array}$$

Solution

$$3D = 9$$

$$D = 3$$

$$3C = ?8$$

$$3C = 18$$

$$C = 6$$

$$B + A + 1 = ?9 \text{ or } 9$$

$$B + A = 8 \text{ or } 18$$

$$\text{If } B = 1$$

$$A = 7$$

$$A = 7, B = 1, C = 6, D = 3$$

b)

$$\begin{array}{r} \\ \\ + \\ \hline 2 A A C \end{array}$$

Solution

$$3C = ?C$$

$$\text{If } C = 5 \rightarrow 3C = 15$$

$$A + B + 1 = 1A \text{ --- (1)}$$

$$C + C + 1 = A \text{ -- Since } C + C = 10$$

$$\text{and } A + 1 = 2$$

$$\text{Then } A = 1$$

$$\text{Sub } A \text{ into eqn (1)}$$

$$A + B + 1 = 1A$$

$$1 + B + 1 = 11$$

$$B = 11 - 2 = 9$$

$$A = 1, B = 9, C = 5$$



4. Alice's house number is 4-digit number. When she moves the first digit to the ones place, she notices the new 4-digit number is bigger than her house number by 4707. What is her house number?

Solution

Let her house number be ABCD. Her new house number is BCDA.

$$BCDA - ABCD = 4707 \text{ ----- (1)}$$

$$1000B + 100C + 10D + A - 1000A - 100B - 10C - D = 4707$$

$$900B + 90C + 9D - 999A = 4707$$

Simplify by dividing both sides by 9

$$100B + 10C + D - 111A = 523$$

$$\text{If } A = 1, B = 6$$

$$600 + 10C + D - 111 = 523$$

$$D = 4$$

$$10C - 10 = 20$$

$$10(3) - 10 = 20$$

$$C = 3$$

$$A = 1, B = 6, C = 3, D = 4$$

Her house number is 1634.



13. Writing Equations

