

Learn how to solve problems that calculators can't

$$999\ 999 \times 222\ 222 + 333\ 333 \times 333\ 334 = ?$$

Computing Large Whole Numbers
Math Olympiad Training

SUPPAHUB
LEARNING NEW SKILLS



The Concept

$$\begin{aligned}1234 &= 1000 \times a + 100 \times b + 10 \times c + 1 \times d \\&= 1000 \times 1 + 100 \times 2 + 10 \times 3 + 1 \times 4 \\&= 1000 + 200 + 30 + 4\end{aligned}$$

Try: 543

$$\begin{aligned}543 &= 100 \times 5 + 10 \times 4 + 3 \\&= 500 + 40 + 3\end{aligned}$$



Example 1 – Simplification

$$\begin{aligned} &999\,999 \times 222\,222 + 333\,333 \times 333\,334 = ? \\ &= (333\,333 \times 3 \times 222\,222) + (333\,333 \times 333\,334) \\ &= 333\,333 \times [(3 \times 222\,222) + (333\,334)] \\ &= 333\,333 \times (666\,666 + 333\,334) \\ &= 333\,333 \times 1\,000\,000 \\ &= 333\,333\,000\,000 \end{aligned}$$



Simple subtraction tips

$$2200 - 22$$

$$= 2178$$

$$222\ 000 - 222$$

$$= 221\ 778$$

$$2222\ 0000 - 2222$$

$$= 2221\ 7778$$



Simple subtraction tips

$$3300 - 33$$

$$= 3267$$

$$333\ 000 - 333$$

$$= 332\ 667$$



Example 2 – Find sum of digits

Find the sum of all digits of $\underbrace{333\dots333}_{2008 \text{ 3s}} \times \underbrace{666\dots666}_{2008 \text{ 6s}}$.

$$333\dots333 \times 666\dots666$$

$$= 333\dots333 \times 3 \times 222\dots222$$

$$= 999\dots999 \times 222\dots222$$

$$= \underbrace{(1000\dots000 - 1)}_{2008 \text{ 0s}} \times \underbrace{222\dots222}_{2008 \text{ 2s}}$$

$$= 222\dots222 \text{ 000}\dots\text{000} - 222\dots222$$

$$= \underbrace{222\dots222}_{?? \text{ 2s}} \text{ 1 } \underbrace{777\dots777}_{?? \text{ 7s}} \text{ 8}$$

$$2 + 7 = 9 \text{ (How many pairs of 9?)}$$

$$1 + 8 = 9 \text{ - one more here}$$

$$2008 \times 9 = 18\,072$$



Example 3 – using Algebra

The sum of three digits is 21.

Digit in the ones place is greater than the tens place.

A new number is 198 more than the original one, is formed by interchanging the digit in the ones place with the digit in the hundreds place.

What is the original number?

Solution

1. Represent the 3 digit number as “abc”.
2. Guess the number $\rightarrow abc = ?$ 678 – is it only answer?
3. Equation (1) : $100a + 10b + c = abc$
4. Equation (2) : $100c + 10b + a - abc = 198$ (after interchanging)

5. Solve the equations:

$$100c + 10b + a - (100a + 10b + c) = 198$$

$$99c - 99a = 198$$

$$99(c - a) = 198$$

$$c - a = 198 / 99$$

$$c - a = 2$$

Try – 759?

$$957 - 759 = 198$$



Example 4 – Application of Algebra

Miss C was born on the 1st January many years ago. In 2002, her age was the sum of all the four digits of the year that she was born in. How old was Miss C in 2002?

Solution

1. Express the year she was born in using equation: $1000 + 100a + 10b + c$ since she must be born before year 2000.
2. Her age = sum of digits of the year she was born
3. $2002 - (1000 + 100a + 10b + c) = 1 + a + b + c$
4. Tip = We know she can't be that old so we assumed "a" is 9.
5. $1001 = 101a + 11b + 2c \Rightarrow 1001 - 101 \times 9 = 11b + 2c$
6. $b = (92 - 2c) / 11$
7. Since b must be a whole number – we can guess c has to be 2.

Answer:

Miss C is born in 1982

$$2002 - 1982 = 20 \Rightarrow$$

And she is 20 in 2002





Maths Olympiad

Unleash **The Maths Olympian** In You!

Advanced (Pr 5 & 6, suitable for 11-13 years old)

- **Get into the spirit of Maths Olympiad**
Non-routine questions classified by type that challenge you to think beyond typical solutions
- **Boost your mathematics problem-solving skills**
Learn mind-sharpening skills and methodologies that transcend curricular knowledge
- **Q.E.D. (Quite Easily Done!)**
Adopt and apply elegant and efficient solutions to complex MO-type problems
- **Step-by-step worked solutions**
Engle it and alternative answers to further learning and understanding

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