

$$5x + 6y = \$84.50 \quad \dots(1)$$

$$2x + 3y = \$41 \quad \dots(2)$$

2. Solving Problems Quickly With Algebra

Math Olympiad Training



Revising basic Algebra

1. A problem can be expressed using **Equations**
2. No. of equations \geq No. of variables
3. Let a character represent a variable
4. Units can be used for checking
5. Substitute value into another equation to check



Revising basic Algebra

X cost 2 times of Y

5 of X and 10 of Y cost \$200.

Find X and Y.

Solution

$$X = 2Y$$

$$5X + 10Y = 200$$

$$5(2Y) + 10Y = \$200$$

$$10Y + 10Y = \$200$$

$$20Y = \$200$$

$$Y = \$200 / 20$$

$$Y = \$10$$

$$X = 2 \times \$10 = \$20$$



Example 1 on Replacement / Substitution

5 tables and 18 chairs cost \$594.

The cost of one table is same as cost of 3 chairs.

Find the cost of a table and a chair.

Solution

x = Tables y = Chairs

$$5x + 18y = \$594 \quad (1)$$

$$x = 3y \quad (2)$$

Substitute (2) into (1)

$$5(3y) + 18y = \$594$$

$$15y + 18y = \$594$$

$$33y = \$594$$

$$y = \$18$$

Substitute y into (2)

$$x = 3 \times \$18 = \$54$$



Example 2 - Money

4 basketballs and 6 footballs cost \$460.

A basketball is \$15 more expensive than a football.

How much is each basketball?

How much is each football?

Solution

x – basketball, y – football

$$4x + 6y = \$460 \dots(1)$$

$$x = y + \$15 \dots(2)$$

Substitute (2) into (1)

$$4(y + 15) + 6y = \$460$$

$$4y + \$60 + 6y = \$460$$

$$10y = \$400$$

$$y = \$40$$

Substitute y into (2)

$$x = \$40 + \$15$$

$$x = \$55$$



Example 3 - Elimination

5kg of sugar and 6kg of rice cost \$84.50

2kg of sugar and 3kg of rice cost \$41

How much is 1 kg of sugar? How much is 1kg of rice?

Solution

x – price of sugar per kg

y – price of rice per kg

$$5x + 6y = \$84.50 \quad \dots(1)$$

$$2x + 3y = \$41 \quad \dots(2)$$

Eqn (2) x 2

$$4x + 6y = \$82 \quad \dots(3)$$

Eqn (1) – Eqn (3):

$$x = \$2.50$$

$$3y = \$41 - 2x$$

$$y = (\$41 - 2x) / 3$$

$$= (\$41 - 2(2.50)) / 3$$

$$= \$36/3$$

$$= \$12$$



Example 4 - Substitution

If 10 rabbits can be exchanged for 2 goats, 9 goats be exchanged for 3 cows and 8 cows be exchanged for 2 horses.

How many rabbits can 5 horses be exchanged for?

Solution

r – rabbits, g – goats

c – cows, h – horses

$$10r = 2g \dots(1)$$

$$9g = 3c \dots(2)$$

$$8c = 2h \dots(3)$$

$$g = 5r$$

Sub into equ (2)

$$9(5r) = 3c$$

$$3c = 45r$$

$$c = 15r$$

Sub into eqn (3)

$$2h = 8c$$

$$h = 8(15r) / 2$$

$$= 60r$$

$$5h = 60r \times 5 = 300r$$

5 horses can be exchanged for 300 rabbits.



Example 5 - Speed

It takes Mark a total of 25 minutes to run a distance of 1000m and then walk another 2000m.

It also takes him 25 min to run a distance of 4000m and then walk another 500m.

How fast can Mark run?

Solution

r = speed of Mark's running (m/min) $\Rightarrow x = 1/r$ (min/m)

w = speed of Mark's walking (m/min) $\Rightarrow y = 1/w$ (min/m)

$$1000/r + 2000/w = 25 \dots(1) \Rightarrow 1000x + 2000y = 25 \dots (1)$$

$$4000/r + 500/w = 25 \dots(2) \Rightarrow 4000x + 500y = 25 \dots(2)$$

Multiply eqn (2) by 4

$$16000x + 2000y = 100 \dots(3)$$

Eqn (3) – Eqn (1)

$$15000x = 75$$

$$x = 75/15000 \text{ min/m}$$

$$= 1/200 \text{ min/m}$$

$$r = 200 \text{ m / min}$$



2. Solve by Comparison & Replacement

